QN. → To find the equation of the circle which cuts the three given circles orthogonally.

Ans. → Let the eqn. of three given circles be

$$x^2+y^2+2g_1x+2f_1y+c_1=0 \quad \text{--- (1)}$$

$$x^2+y^2+2g_2x+2f_2y+c_2=0 \quad \text{--- (2)}$$

$$x^2+y^2+2g_3x+2f_3y+c_3=0 \quad \text{--- (3)}$$

Let $x^2+y^2+2gx+2fy+c=0$ --- (4) be that circle which cuts the given three circles orthogonally

$$\therefore 2gg_1+2ff_1-c-c_1=0 \quad \text{--- (5)}$$

or, $2gg_2+2ff_2-c-c_2=0$ --- (6)

$$2gg_3+2ff_3-c-c_3=0 \quad \text{--- (7)}$$

Now, eliminatry g, f from (4), (5), (6), (7) we have,

$$\begin{vmatrix} x & y & x^2+y^2+c \\ g_1 & f_1 & -c-c_1 \\ g_2 & f_2 & -c-c_2 \\ g_3 & f_3 & -c-c_3 \end{vmatrix} = 0$$

required condition

Theorem: 7

QN. – To prove that the three radical axis of three coplanar circles taken in pairs are concurrent provided that their centres are non-collinear.

or, QN. → Prove that the radical axis of three circles taken in pairs are concurrent.

Ans. → Let the eqn. of three circle be

$$S_1 = 0 \quad \text{— (1)}$$

$$S_2 = 0 \quad \text{— (2)}$$

$$S_3 = 0 \quad \text{— (3)}$$

Now, Radical axis of (2) and (3) is

$$S_2 - S_3 = 0 \quad \text{— (4)}$$

and the radical axis of (3) and (1) is

$$S_3 - S_1 = 0 \quad \text{— (5)}$$

and the radical axis of (1) and (2), we have

$$S_1 - S_2 = 0 \quad \text{— (6)}$$

$\therefore$ (4) + (5) + (6) = 0

Hence, the radical axis of three circle taken in pairs are concurrent.

Theorem: 8

QN. → Define co-axial circle and obtain the equation of a system of co-axial circles in the simplest form.

Q. Defin Co-axial circle and Find the equation of a system of Co-axial circles in the simplest form.

Ans:→ Co-axial-circle:– A system of circle is said to be Co-axil when they have the same ratical axis i.e. the radical axis of any pair of circle is the same.

Eqn. of system of Co-axial circle in simplest form:–

Let the eqn. of two circle, be

$x^2+y^2+2g_1x+2f_1y+c_1=0$ — ① and

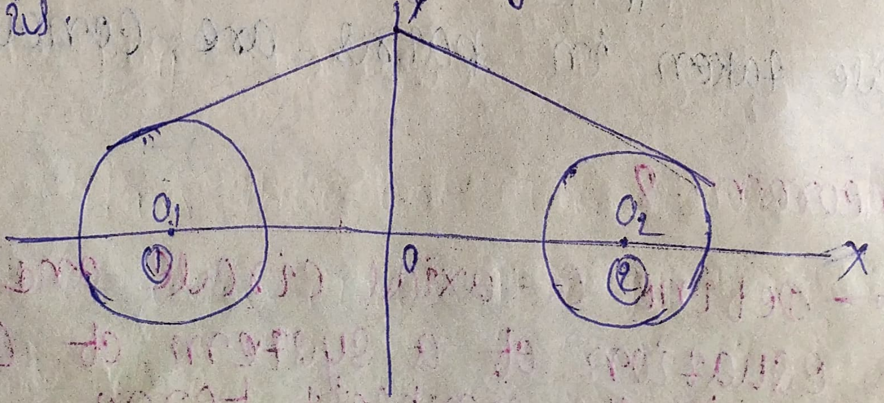
$x^2+y^2+2g_2x+2f_2y+c_2=0$ — ②

Hence, the radical axis of ① and ② is

$2x(g_1-g_2)+2y(f_1-f_2)+c_1-c_2=0$ — ③

Since the radical axis is ⊥ to the line Joining their centres of any pair of circles of the system.

Let us choose common Radical axis as Y axis and line of Joining their centre as x-axis.



the y co-ordinate will be zero on the x axis i.e. the line joining the centre of all such circles.

$\therefore f_1 = 0$ and $f_2 = 0$

As we have supposed that y axis as the radical axis.

Hence, $x = 0$

Hence, from eqn. ③

$\therefore C_1 - C_2 = 0$

$\therefore C_1 = C_2 = C$ Let

Hence, eqn. ① and ② takes the form

$x^2 + y^2 + 2g_1x + c = 0$

and $x^2 + y^2 + 2g_2x + c = 0$

~~Hence these circle can the represented~~

~~by $x^2+y^2+2gx+c=0$~~

Similarly: other co-axial circle of the above type is

$x^2 + y^2 + 2g_3x + c = 0$

Hence, the General eqn. of a system of co-axial circle is $x^2 + y^2 + 2gx + c = 0$ when g is variable and c = constant.

Theorem: 9

QN: → Define limiting point of a system of co-axial circle, If L and L' be the limiting point of a system of co-axial circles: The

Polar of either of these points with respect to any circle of the system passes through the other.

Ans.→ <u>Limiting points of a system of co-axial circle</u> :— Limiting point of a system of co-axial circle are the center of the point circle, i.e. circle of zero radius of the system of circle or the family of circle.

As the eqn. of the co-axial circle is $x^2+y^2+2gx+c=0$ — (1)

or, $x^2+2gx+y^2+c=0$

or, $x^2+2gx+g^2-g^2+(y-0)^2+c=0$

or, $(x+g)^2+(y-0)^2=g^2-c=(\sqrt{g^2-c})^2$ — (2)

∵ the radius of co-axial circle is

$$\sqrt{g^2-c}$$

for circle of zero radius

$$g^2-c=0$$

$$\therefore g^2=c$$

$$\therefore g=\pm\sqrt{c}$$

Thus there are two circles of zero radius of co-axial circle. Putting $g=\pm\sqrt{c}$ in eqn. (2) the eqn. of circle of zero radius $(x\pm\sqrt{c})^2+(y-0)^2$

Hence centeres of such